

A Study on Sixth-grade Turkish Students' Spatial Visualization Ability

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The aim of this study is to evaluate sixth-grade Turkish students' spatial visualization ability and to determine the strategies that students employ and the mistakes that they make while solving the problems requiring spatial reasoning. Therefore, the study examined the following achievements: visualizing different view images of three-dimensional structures made from unit cubes, interpreting two-dimensional pictorial representations of three-dimensional objects and finding the faces of the cube. In this study, spatial visualization behaviors were limited to the achievements covered by the Turkish primary education mathematics curriculum. Data were collected by means of the Block of Cubes Test, which was designed to measure student achievements about the ability to mentally visualize different-perspective views of three-dimensional structures made from unit cubes, to interpret two-dimensional pictorial representations of three-dimensional structures and to find the visible faces of the cube. The findings were based on the scores received by a total of 60 participant students from this test and the clinical interviews conducted with 21 students chosen randomly among the participants in order to identify the challenges experienced by them in the test.

One definition of spatial ability is the combination of abilities to visualize spatial objects, recognize their different perspective views, and move them as a whole or their parts separately. Research on spatial ability has significant implications for science, geometry, engineering and architecture. Evidence suggests that spatial ability is closely associated with mathematics achievement (Battista, 1990;

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Fennema & Sherman, 1977; Guay & McDaniel, 1977). Similarly, Battista, Wheatley, and Talsma (1989) report a significant positive correlation between spatial ability and problem solving performance.

Spatial ability is significant for teaching many topics in mathematics, particularly geometry. Learning about space and developing spatial abilities (such as drawing, producing models, making revisions on models and manipulating the environment) originate primarily from geometric considerations. There exists a need for mathematics instruction to develop students' perception of space and their spatial intelligence since spatial reasoning is required for understanding, interpreting and recognizing geometric world, according to the National Council of Teachers of Mathematics (NCTM; 1989, p.48). Many interrelated studies point out that spatial ability is associated with mathematical achievement. (e.g., Fennema & Sherman, 1977; McKee, 1983; Battista, Wheatly & Talsma, 1989; Keller, Wasburn-Moses & Hart, 2002). Spatial thinking is significant for many mathematical procedures such as forming simple geometric shapes, forming more complicated geometric shapes out of simple ones, solving puzzles about space and numeral concepts, and learning about the relations between numerals. NCTM (1989) reported that students need to employ mental visualization, logical reasoning and geometric models so that they can solve problems throughout their academic career. On the other hand, evidence suggests that primary school students (Battista & Clements, 1996; Ben-Chaim, Lappan, & Houang, 1985; Olkun, 2003a) and even high-school students (Hirstein, 1981; Olkun, 2003a) have difficulty finding the number of unit cubes in rectangular buildings made from small cubes. In a study about the types of the errors made by students (of varying ages) while finding the number unit cubes in prisms, Hirstein (1981) claims that students make errors about "visible cubes or the faces of cubes" and therefore they confuse volume with the surface area. Also, Ben-Chaim, Lappan and Houang (1985) reported that the students in their study made mistakes while determining the number of the edges and corners of the prisms given. This includes counting the edges and corners more than once. As a result of these mistakes, the authors concluded that

students had difficulty in visualizing the pictorial representations of three-dimensional buildings and therefore provided wrong answers for this kind of problem. They further argue that students can solve this type of problem by using their spatial visualization ability and spatial reasoning skills.

Mathematics curriculum has recently been revised in Turkey and it now covers several achievements concerning the development of students' spatial ability within the sixth-, seventh-, and eighth-grade geometry learning domains. These achievements are referred to in the 2009 report of Ministry of National Education (MEB; 2009) as

being able to draw different-perspective views of structures made from identical cubes, to form structures based on their different view images with unit cubes and draw them on the isometric paper, to form structures by using multiple cubes based on their drawings, and to draw the different view images of structures that are made from multiple cubes. (p. 172)

These achievements are intended to improve students' spatial ability.

In summary, our aim in this study is to determine the achievement level of primary education sixth-grade students in finding the total number of unit cubes and the number of visible surfaces in different numbers in three-dimensional structures which were formed by using pictorial representations of unit cubes. Therefore, we sought answers to the following questions:

- a) What is the achievement level of primary education 6th grade students in finding the total number of unit cubes in pictorial representations of three-dimensional structures made from unit cubes?
- b) What is the achievement level of primary education 6th grade students in finding the different numbers of visible faces in pictorial representations of three-dimensional structures made from unit cubes?
- c) What is the achievement level of primary education 6th grade students in drawing the different perspective views of three-dimensional structures made from unit cubes?

d) What strategies do students use and what errors do they make while solving this kind of problems?

Method

This study used a mixed method research design consisting of quantitative and qualitative techniques. In the first part of the study, statistical analysis was carried out by collecting quantitative data after administering the Block of Cubes Test (a description of this test is given later). In the second part, the qualitative data were collected by using the clinical interview technique. These interviews were carried out with 21 students who were randomly chosen among the groups taking the Block of Cubes Test. During these interviews, each of the students was asked questions about the problems which they answered in the Block of Cubes Test and how they solved them.

Study Sample

The study sample included a total of sixty sixth-grade students attending two classes in a primary school in Turkey. According to the primary education mathematics curriculum in Turkey, sixth-grade students should be able to do the following: (a) mentally visualize different-perspective views of three-dimensional structures made from unit cubes, (b) interpret two-dimensional pictorial representations of three-dimensional structures and (c) find the visible faces of the cube. At the end of the school year, a randomly-chosen group among these sixth grade students was administered the Block of Cubes Test in order to test their spatial ability (within the limits of the achievements covered by the current sixth-grade curriculum). The students were then divided into three groups. These groups were labeled achievers, average-achievers and underachievers based on the number of their correct answers on the test. At the qualitative stage of the study, seven voluntary students were chosen in each achievement group. Therefore, we interviewed a total of 21 students to determine their strategies and mistakes in solving problems requiring spatial reasoning. In qualitative research, researchers are generally recommended to change their participants' names by

pseudonyms in order to conceal the participants' identities (Patton, 2002). During the analysis of clinical interviews, the participants' real names were not used; instead, the participants' names were coded as S1 to S21, and the researcher was coded as R.

Data Collection Tools

In this study, data were collected through the Block of Cubes Test, which was administered to 60 students, and clinical interviews, which were conducted with 21 volunteers who were chosen among those taking the test.

The Block of Cubes Test

The Block of Cubes Test was designed in accordance with the Turkish primary education mathematics curriculum to determine the students' spatial ability level. To do this, the test is used to examine how well students can mentally visualize different-perspective views of three-dimensional structures made from unit cubes, interpret two-dimensional pictorial representations of three-dimensional structures and find the visible faces of the cube. The questions were taken from the Block of Cubes Test in Spatial Reasoning e-book retrieved from www.wrightgroup.com/download/cp/g6_geometry.pdf.

The test was simultaneously applied with 60 students in different classes for one hour. It was evaluated separately for each of the students. The test results were coded in three groups as unanswered, incorrect and correct and then their percentages were calculated. The error analyses based on their missing or incorrect answers in the student answer sheets are presented in the figures in the Findings section.

The Block of Cubes Test consists of 42 questions including 7-item sets prepared for each of the 6 blocks. The test was prepared in light of the opinions of experts and sixth-grade mathematics teachers. As these kinds of questions have frequently been used in previous studies (Battista & Clements, 1996; Ben-Chaim, et al., 1985), a validity or reliability study was not carried out (Olkun, 2003a). The actual questions from the Block of Cubes Test are given in the Appendix.

Clinical Interview

The clinical interview method can be defined as face-to-face interviews between the researcher and the individual (respondent) in which the interviewer asks the interviewee questions in order to investigate a specific subject and obtain particular information (Kaptan, 1993).

The main purpose of clinical interviews is to determine the structure and extent of the individual's knowledge about a particular subject (Zaskis & Hazan, 1999). Because of this feature, this method has often been used in mathematics education and research into the students' ways of thinking.

This study focused on cognitive processes such as explaining and making inferences as well as evaluation of the students' spatial visualization ability. Finding out the students' thoughts when they were asked questions was necessary to investigate these cognitive processes. The clinical interview technique was considered to be appropriate for this aim. Video and audio recordings of the clinical interviews were made. Each interview session lasted approximately 30 minutes. The clinical interview data were analyzed by combining direct quotations from the students' remarks during the interviews with observation notes of the researcher.

Findings and Interpretation

This section presents the findings obtained regarding the aim of the study. First, the answers to each question are shown in percentages for the 6 different blocks of the test and then those percentages are interpreted.

Findings and Interpretations about the Block of Cubes Test

The results for the test question "How many cubes are in the block?" are given in Figure 1.

As is shown in Figure 1, the vast majority of the students gave incorrect answers for the first question, which asked for the number of the unit cubes in three-dimensional structures. The students gave the highest number of correct answers for

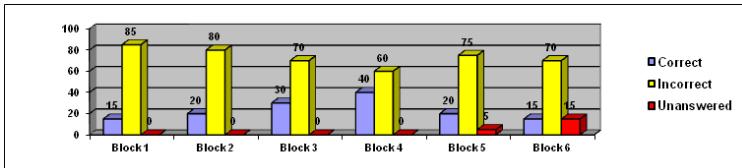


Figure 1. Results on each block for the test question “How many cubes are in the block?”

block 4. This could be explained by the fact that Block 4 is considered less complicated than the other blocks since it contained less unit cubes and there is symmetry (see Figure 18). Thus, we conjecture it was easier for the students to see and count the unit cubes of Block 4.

The results for each block for the test question “How many cubes in the block have 4 visible faces?” are given in Figure 2.

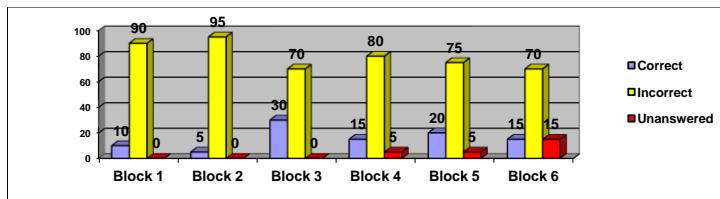


Figure 2. Results on each block for test question “How many cubes in the block have 4 visible faces?”

Figure 2 shows that the great majority of the students gave incorrect answers and performed incomplete counting for the question concerning the number of cubes having 4 visible faces in three-dimensional structures made from unit cubes. This low level of achievement might be associated with the fact that the students had difficulty in visualizing three-dimensional structures and could not properly understand the pictorial representations of the blocks. The students gave the most correct answers for Block 3.

The results on each block for the test question “How many cubes in the block have 3 visible faces?” are given in Figure 3. According to Figure 3, none of the students gave correct answer to the third question. This might be attributed to the fact that the students did not take the hidden faces into consideration or they forgot to count the cubes remaining at the

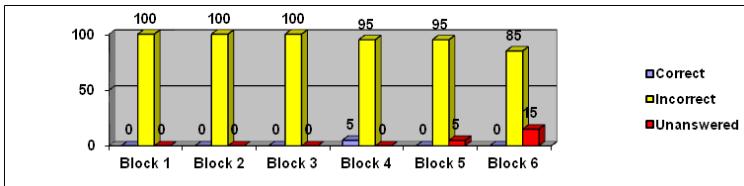


Figure 3. The results on each block for the test question “How many cubes in the block have 3 visible faces?”

rear while mentally rotating or answering by looking at the drawing.

The results for each block for the test question “How many cubes in the block have 2 visible faces?” are given in Figure 4.

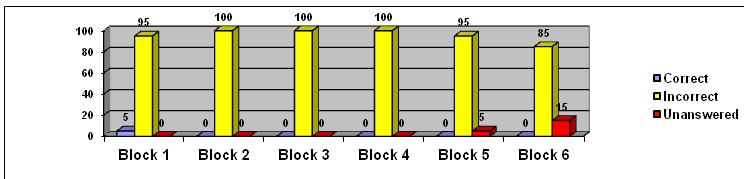


Figure 4. The results for each block for the test question “How many cubes in the block have 2 visible faces?”

As shown in Figure 4, none of the students gave correct answers to this question for any of the blocks except for a small number of correct answers for Block 1. This situation might be associated with the fact that the cubes with 2 visible faces were in the hidden parts of the block represented on paper and that the students did not take this fact into consideration. Therefore, it could be suggested that the students couldn't mentally and precisely visualize the pictorial representations in three-dimensions. The students' answer sheets indicated that those students who gave incorrect answers performed incomplete counts. Perhaps some of the students were not capable of mentally rotating the block properly. Another cause of the students' incorrect answers might be their lack of concentration (which is an important factor in such problems) while attempting to solve this question.

The results for each block for the test question “How many cubes in the block have no visible faces?” are given in Figure 5.

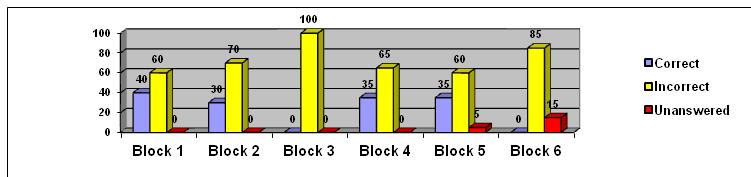


Figure 5. The results for each block for the test question “How many cubes in the block have no visible faces?”

According to Figure 5, none of the students were able to provide correct answers to the question for Blocks 3 and 6. On the other hand, the percentage of the correct answers for Blocks 4 and 5 is 35%. Thus, the vast majority of the students also gave incorrect answers to this question. This might be attributed to their inability to perform mental rotations of three-dimensional structures made from unit cubes and difficulty in visualizing the blocks. The levels of achievement of Block 1, 4 and 5 are almost equal but the highest level of achievement was found in these Blocks. Perhaps this situation was caused by the students’ success in mentally rotating these blocks more easily and recognizing that there were no cubes without a visible face.

The results for each block for the test question “How many cubes in the block have 1 visible face?” are given in Figure 6.

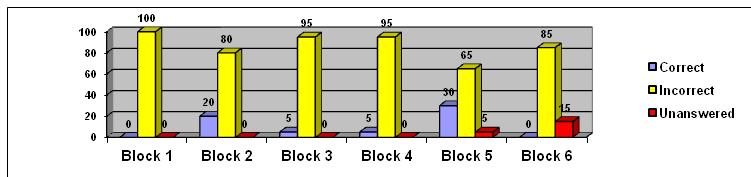


Figure 6. The results for each block for the test question “How many cubes in the block have 1 visible face?”

Figure 6 indicates that none of the students gave correct answers to the test question for Blocks 1 and 6. On the other hand, Block 5 received the highest percentage of correct answers with 30%. The reason for this low level of achievement might be attributed to the fact that there were too many cubes with one visible face whose faces were hidden in pictorial representations but did exist in three-dimensions.

Also, another reason may be the fact that students couldn't properly visualize the blocks in three-dimensional space.

The results for each block for the test question asking the students to perform drawings of different view images of the blocks are given in Figure 7.

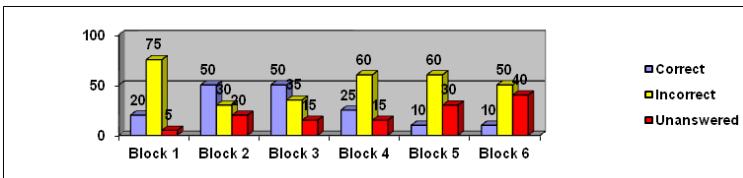


Figure 7. The results for each block for the test question asking the students to perform drawings of different view images of the blocks.

As shown in Figure 7, the percentage of the correct answers for each block for the last test question was relatively high in comparison with the other test questions. We conjecture this is because this question is easier to comprehend for the students. However, the fact that students made mistakes in their drawings might be associated with their inability to perform a spatial reasoning about the three-dimensional structures given with pictorial representations and to visualize them mentally.

Findings and Interpretations about the Clinical Interviews

21 students were selected for clinical interview according to the Block of Cubes Test results (7 students from each level of achievement: achievers, average achievers, and underachievers). These students were then interviewed about the test questions which they answered in the test stage and how they answered them.

The results from the analysis of the interviews of 9 out of the 21 students are given in detail. We choose these nine because their responses represented as many situations as we thought were possible. Samples of the students' responses to the test questions and the interpretations derived from the clinical interviews are presented below.

In this section, the findings are analyzed based on the students' ways of thinking. The first subsection deals with

students who did not know what cubes, faces, etc., were; the second subsection deals with students who only attended to visible cubes because of difficulties in visualization, etc.; the third subsection deals with students who had trouble keeping track of counting; and the fourth subsection deals with students whose main issue was rotating their visualizations to correctly count the numbers of exposed faces. The final subsection discusses student strategies such as the use of symmetry to answer questions.

Students with misconceptions and errors

In the first subsection, we discuss the students who were unaware of the definitions of cube, face, and other similar terms.

The dialogue between the researcher and S1 (in the underachiever group) regarding the questions about the Block of Cubes Test block 1 (see Figure 8) and the researcher's notes are given below.

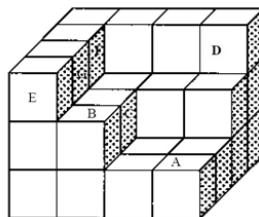


Figure 8. The Block of Cubes Test, block 1.

R: The block above was made from unit cubes. Can you mentally visualize this block?

S1: Yes, as a box, I counted the units inside. The front part is a little bit clear.

R: How many unit cubes are in the block? How did you answer this question?

S1: I counted the squares because I knew that a unit was a square.

R: How did you count? Can you show me?

S1: 1,2,3...30.

R: Do you know what the cube is? Do you know the cube's shape?

S1: The cube is square-shaped.

R: OK. Can you give an example of the cube? Is there any example of the cube shape here?

S1: Well, the TV would be a cube if it didn't have the rear part.

R: Can you show a cube in this shape (*block of cubes*)?

S1: This is a cube; there are 4 sides. These are the same, too. These all have to be cubes because they're rectangular. (*The student regarded one face of the cube as a cube and counted all the visible faces as cubes*).

R: How many cubes in the block have 4 visible faces? What does face of a cube mean? Can you show me a face of the cube?

S1: For example, this is a square, but this one is a face and so is this one... (*The student referred to sides of the square as faces*).

S1: I counted them. 1, 2, 3... 7. This one is half because it's not a complete cube... There are more here (*the student labeled the partly-visible faces as half-cubes*). There are 12 of them. You told us to visualize them in our mind. I thought about them as if I could see them and then counted them. That is how I got it.

R: Well, how many cubes in the block have 1 visible face? What do you get by this question?

S1: You mean a single face (*the student drew an edge of the square*).

R: How did you come up with the number 7?

S1: I counted them. I mean I visualized them.

R: OK. How many cubes in the block have no visible faces?

S1: Err, cubes with no visible faces... I thought they were closed and counted those in this box.

R: Closed? What do you mean by "closed"?

S1: I thought that the whole block was closed, so I counted the ones inside.

R: Which ones?

S1: There are 4 of them.

R: OK. Can you draw the right view image of the block?

S1: I see the block as I drew it (*the student the right view image as stairs*).

The right view image of Block 1 in the Block of Cubes Test (see Figure 8), drawn by S1, is given in Figure 9.

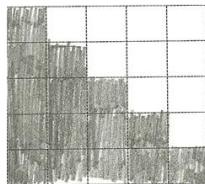


Figure 9. The right view image of Block 1 drawn by S1.

Not recognizing and knowing the definition of the cube, the student S1 considered that the visible faces (quadrangles seen in the block) were cubes. Therefore, when asked for the number of cubes in the block, the student gave an answer by counting all the visible faces. When asked about the definition of “face” and to show a face of the cube, the student pointed at one edge of a square and labeled it as an example of face. Evidently, being confused about the cube, the face and the edge, the student gave incorrect answers to all of the questions concerning the block. When asked to draw the right view image of the block, the student focused on the cubes on the right end of the block and came up with an incorrect drawing in the form of stairs. Also, the student ignored the number of cubes in this drawing.

The dialogue between the researcher and S14 (underachiever) regarding the questions about the Block of Cubes Test block 5 (see Figure 10) and the researcher's notes are given below.

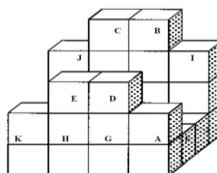


Figure 10. The Block of Cubes Test Block 5.

R: This block is composed of unit cubes. How many unit cubes are in this block? How did you answer this question?

S14: I counted them all (*the student counted all of the visible faces*). I didn't count here because there are no cubes here (*the student didn't count the partly-visible faces*). But, it's 26...

R: Do you think you forgot to count any part?

S14: Yes, I guess so.

R: Well, how many cubes in the block have 4 visible faces? What does this question mean?

S14: I answered it as none because I don't have the slightest idea about what it means.

R: What does "face" mean? Which part of the cube does it refer to?

S14: Parts like this one (*the student pointed at a face correctly*).

R: OK then. How many cubes in the block have 3 visible faces?

S14: I don't know.

R: Can you draw the left view image of the block?

S14: I'm really sorry, but I cannot remember.

S14 was not able to recognize cubes. This student counted the quadrangles while trying to find the number of cubes in the block but didn't count partly-visible quadrangles, saying that "they cannot be cubes". When asked about the definition of the face, the student gave a correct example of it. However, the student couldn't answer the questions about the numbers of visible faces because of not recognizing the cube and not visualizing it mentally. Also, the student failed to draw the left view image of the block.

Students Attending only to Visible Cubes

In the second subsection, the students who attended only to visible cubes because of difficulties in visualization are discussed. The dialogue between the researcher and S3 (average achievers) regarding the questions about the Block of Cubes Test block 1 (see Figure 8) and the researcher's notes are given below.

R: This block is made from unit cubes. Can you show me the cubes in this block?

S3: Here (*the student pointed at one of the cubes*).

R: So, how many cubes are in this block? And, how did you answer this question?

S3: I answered the question by visualizing the cubes in that space in my mind as well.

R: Why are you trying to imagine the cubes in that space?
There are no cubes there. Now, let's look at the block.
Does the block look like anything you know?

S3: The square, I guess.

R: Which parts look like the square?

S3: ...(*silence*)

R: Do you think this block looks like any furniture you know?

S3: Oh yes. It looks like an armchair.

R: OK. Now, how many cubes are in this block? And, how did you answer this question?

S3: I counted them.

R: How did you make the counting?

S3: There are 19 of them (*the student counted the visible cubes one by one*).

R: How many cubes in the block have 4 visible faces?

R: And, what does "face" mean? Can you show me a face of the cube? (*The student pointed at one of the faces of Cube A correctly*).

R: Let's count the cubes with 4 visible faces. (*The student counted the visible faces of Cube A and told the cube had 4 visible faces*).

R: You will imagine that you are holding the block in your hands and you can see the rear part. Are there any other cubes with 4 visible faces like this one?

S3: There's one here (*the student marked Cube D*) and here (*the student marked Cube E*).

R: Are there any more? Why don't you try counting? Do you think this one may be one of them? (*The researcher marked one of the cubes randomly*).

S3: Um, 1,2,3... No. Not this one. I could find just 3.

R: Alright. How many cubes in the block have 3 visible faces? We've already found one of them. (*The researcher pointed at the cube with 3 visible faces which the student had already found by counting*).

S3: 1,2,3,4,5 and 6; there are 6 of them.

R: OK. Are there any cubes in the block with no visible faces at all?

S3: There aren't any.

R: Why do you think so?

S3: I always see at least one of their faces when I look at them.

R: So, can you draw the right view image of the block? How would you see the block if you looked at it from the right?

S3: It looks like stairs...*(The student drew the right view image of the block as steps of stairs; see Figure 11)*

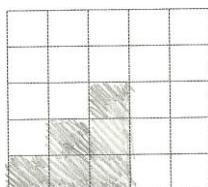


Figure 11. The right view image of Block 1 drawn by S3.

S3 recognized the concept of cube and knew about faces, edges, and corners. However, when asked the number of cubes in the block, the student was unable to think about the hidden cubes and gave an answer by counting just the visible cubes in the block. Perhaps the student couldn't mentally visualize the block and gave incorrect answers to the questions about the different numbers of visible faces. Not visualizing the block properly, the student described the block as "stairs" and drew it in an incorrect way. We conjecture that this was probably due to the fact that the student couldn't produce two-dimensional representations of an object visualized mentally in three-dimensions.

The dialogue between the researcher and S8 (underachievers) regarding the questions about the Block of Cubes Test block 2 (see Figure 12) and the researcher's notes are given below.

R: So, how many cubes are in this block, which is made from unit cubes? How did you answer this question?

S8: I counted them this way: 1,2,3...21, and 22 *(the student counted the cubes in the block marking them one by one but did not think about the hidden ones)*.

R: And now, let's see the last question. You're supposed to draw the left view image of the block? How would you see the block if you were standing on the left of the block?

S8: This is how I would see it (*the student drew the left view image of the block incorrectly; see Figure 13*).

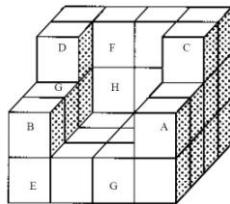


Figure 12. The Block of Cubes Test block 2.

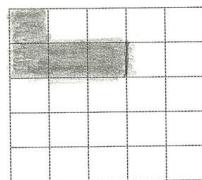


Figure 13. The left view Image of block 2 drawn by S8.

The student counted only the visible cubes by giving them numbers to find the total number of cubes. In addition, when asked to draw the image of the block from the left, the student drew a similar representation by looking the right sight of the block but failed to place the faces correctly on paper. The student's answers to the questions about the other blocks were incorrect due to failure to count the hidden faces.

Students with Trouble Keeping Track of Counting

In the third subsection, the students who had trouble keeping track of counting are discussed. The dialogue between the researcher and S2 (average achiever) regarding the questions about the Block of Cubes Test block 1 (see Figure 8) and the researcher's notes are given below.

R: Let's look at the block 1. Does the block look like anything you know?

S2: It looks like an armchair.

R: The question asks the number of cubes in the block. How did you answer this question?

S2: I answered the questions very quickly during the test. Maybe I made a mistake.

R: It's OK. You don't have to hurry now.

S2: There are 1,2,3,4 at the bottom and 1,2,3,4 in the front.

So... 4 times 4 make 16. There are 16 cubes at the bottom (*the student counted the cubes in the right at the bottom of the block and the other 4 in the bottom right*).

R: You can write them down if you like so that you won't forget them. You found 16 cubes at the bottom.

S2: Then comes the second row. 1,2,3,4 and 5 (*the student counted the visible cubes in the front*). There are cubes in the rear, too (*the student pointed at the cubes remaining in the rear part of the block and counted them*). Er, there are 1,2,3 cubes and more, 4,5,6,7...

S2: I counted 9 just a minute ago, but I'm not sure now.

R: Alright. Take your time.

S2: (*the student showed the cubes in the rear part*) There are 4 more here (*the cubes in the second row in the rear*). And there are 4 more here, too. (*the student showed the cubes in the second row in the left end*). There are 8 cubes in total in the rear part, and one more in the front; that makes 9,10,11,12 and 13.

S2: Oh, we have the upper row, too. I'm counting them... 1,2,3,4,5,6 and 7. There are 36 cubes in total.

R: Hm, the last question asks you to draw the right view image of the block. Suppose that you look at the block from the right; how would you see it?

S2: We see this part first (*the student pointed at the faces of the cubes in the right end*). Then these 2 faces complete here (*the student pointed at the right faces of Cubes B and C in the second row*). And finally, these complete here (*the student pointed at the right faces of Cubes E, G, and F in the top row*).

R: Can you draw it?

S2: We have these 2 here and these 3 here (*the student drew the right view image of the block correctly; see Figure 14*).

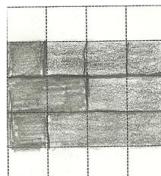


Figure 14. The Right View of Block 1 drawn by S2.

S2 understood the definition of cube and knew about faces, edges, and corners. However, the student did not realize that he counted the cube in the rear left corner twice while counting the cubes in the second row. On the other hand, the student did not make any mistakes in drawing the right view image of the block.

The dialogue between the researcher and S4 (average achiever) regarding the questions about the Block of Cubes Test block 2 (see Figure 12) and the researcher's notes are given below.

R: This block is made from cubes. So, how many cubes are in this block? How did you answer this question?

S4: By counting.

R: Can you count again?

S4: 1,2,3,...9,10,...13,14,... and 2 more below and then 15,16,...30 and 31 in total.

R: But you found 35 before... in the test. Are you sure?

S4: Yes. I'm sure... Wait a minute, there are 3 more here, so it's 34 now (*the student counted the 3 cubes in upper left corner once more*).

R: Do you know what "face" means? Can you show me a face of the cube?

S4: Here (*the student pointed at one of the faces of a cube correctly*).

R: OK. Can you draw the left view image of the block? How would you see the block if you looked at it from the left?

S4: Like this (*the student drew the left view image but added one more cube which shouldn't be there; see Figure 15*).

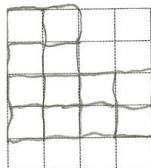


Figure 15. The left view image of Block 2 drawn by S4.

S4 understood cubes and was able to recognize faces, edges, and corners. When asked to give the number of cubes in the block, the student was able to think about the hidden cubes but counted incorrectly due to trouble keeping track. We conjecture that perhaps the student couldn't mentally visualize the different views of the block and gave incorrect answers. These mistakes were repeated in the other blocks as well. Also, the student gave different and more correct answers during the interview than in the test. It is possible that the student tried to be more careful in the interview and had more time to think about the answers to the questions. In addition, the student drew the left view image of the block almost correctly with just one extra cube. This might have been caused by the student's trouble keeping track of counting.

The dialogue between the researcher and S20 (underachiever) regarding the questions about the Block of Cubes Test block 6 (see Figure 16) and the researcher's notes are given below.

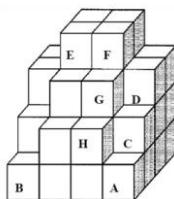


Figure 16. The Block of Cubes Test block 6.

R: How many unit cubes are in this block, which is made from unit cubes? How did you answer this question? Do you know the cube shape?

S20: Yes. This is a cube for example (*the student correctly pointed at one of the cubes*).

R: OK. Go on.

S20: I counted the cubes. 1,2,3... 9 here (*the student counted the cubes in the right part of the block*). Then there are more... 10,11,12,...22 and 23 in total (*the student counted the visible faces*).

R: But you found the number as 35 before.

S20: Hm, yes... there must be more cubes under these (*the student pointed at the hidden cubes*). So, there should be 24,25...30 and 31 in total.

R: Sorry, I didn't understand how you counted.

S20: OK. Let me count again.

S20: Um, I'm confused now.

R: No problem. Let's see the next question. Can you draw the top view image of the block?

S20: I would see the block as stairs from the top, so I could see here (*the student could see the top faces of the cubes; see Figure 17*).

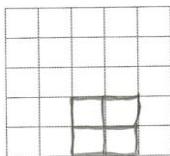


Figure 17. The top view image of block 6 drawn by S20.

S20 knew about cubes but couldn't distinguish faces from edges. While trying to find the number of unit cubes, the student first counted the visible cubes and then noticed the hidden cubes but failed to count them in a systematic way. The student couldn't mentally visualize the block properly and thus was unable to obtain the correct number of unit cubes. Similarly, the student made errors about the numbers of the visible faces of the cubes in the rear part. In addition, the student came up with an incorrect top view image drawing of the block.

Students with errors in counting visible faces

In the fourth subsection, the students whose main issue was rotating their visualizations to correctly count the numbers of exposed faces are discussed. The dialogue between the

researcher and S2 (average achievers) regarding the questions about the Block of Cubes Test block 1 (see Figure 12) and the researcher's notes are given below.

R: OK. How many cubes in the block have 4 visible faces?

S2: None. Some cubes have 1,2,3 faces (*the student pointed at Cube B with 3 visible faces*).

R: Imagine that you are holding this block in your hands. You can rotate it.

S2: Um, in that case, we get this (*the student marked Cube D*). And this, too (*the student marked Cube D*). There are 2 of them.

R: Now. How many cubes in the block have 3 visible faces?

S2: 1,2,3...

R: But, you just counted this as a cube with 4 visible faces (*the researcher pointed at Cube E*).

S2: That's right. There are 1,2,3...9,10 of them. And there are more in the corners.

R: OK. How many cubes in the block have no visible faces at all?

S2: None.

R: None?

S2: Yes, none. All of them have visible faces.

S2 seemed to understand the concepts of cube, face, edge, and corner. Despite following the right path to answer the questions, the student was careless at times and did not concentrate on the different view images of the block, so he completely skipped the hidden faces of the cubes and counted wrong. The student's answers to the other questions in the test showed that he performed inaccurate counting at times.

The dialogue between the researcher and S4 (average achievers) regarding the questions about the Block of Cubes Test block 2 (see Figure 12) and the researcher's notes are given below.

R: OK. How many cubes in the block 2 have 4 visible faces?

S4: We have this one (*the student marked Cube A*), then here (*the student marked Cube B*), here (*Cube C*) and this one (*Cube D*). There are 4 in total.

R: How many cubes in the block have 3 visible faces?

S4: Again this one (the student marked Cube A).

R: But you just said it had 4 visible faces.

S4: Yes. That's right. This one has 3 visible faces: 1,2, and 3.

And that one too: 1,2,3.. so, there are 2 of them.

R: Alright. Let's continue. How many cubes in the block have no visible faces?

S4: None.

R: How many cubes in the block have 1 visible face?

S4: None again.

The student (S4) couldn't mentally visualize the different views of the block and gave incorrect answers to the questions about the different numbers of visible faces. These mistakes were repeated in the other blocks as well.

The dialogue between the researcher and S8 (underachievers) regarding the questions about the Block of Cubes Test block 2 (see Figure 12) and the researcher's notes are given below.

R: How many cubes in the block 2 have 4 visible faces? What do we mean by "face"? Can you show me one of the faces of the cubes in this block?

S8: This part (*the student defined the face correctly by marking the area remaining inside the four edges of a face*).

R: Now, let's count the cubes with 4 visible faces.

S8: OK. 1,2,3,4 ... and 19. There are 19 of them (*Despite recognizing the face, the student considered all the cubes to have 4 visible faces*).

R: Right, how many cubes in the block have 3 visible faces?

S8: 1 and 2 (*counting, the student marked Cubes A and B in the block*). There are 2 of them.

R: How many cubes in the block have no visible faces?

S8: All of them have visible faces, so the correct answer should be none.

S8 was able to visualize a cube but had some misconceptions about faces, edges, and corners. However, when asked the number of cubes with 3 visible faces in the block, the student did not notice the hidden cubes probably due

to inadequate mental visualization. It appeared that the student couldn't visualize the pictorial representations in three-dimension. Also, when asked to say the number of cubes with 4 visible faces, the student counted the quadrangles in the block instead of the cubes. We conjecture this is because of not being able to differentiate the two terms "the face" and "the edge".

The dialogue between the researcher and S11 (average achiever) regarding the questions about the Block of Cubes Test block 4 (see Figure 18) and the researcher's notes are given below.

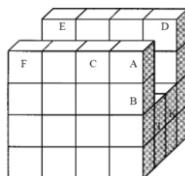


Figure 18. The Block of Cubes Test block 4.

R: So, how many unit cubes are in this block? How did you answer this question?

S11: Well, I first counted these... there are 4 here and 4 more here. And we've got 16 here in total (*the student counted the 16 cubes in the rear part of the block*). There 8 cubes here and 8 more cubes under them. That is 16 in total (*the student counted the cubes in the central part of the block*). There are 16 cubes here already (*the student counted the cubes in the front part*). Therefore, we've got 48 unit cubes in total.

R: Can you show me one of the faces of the cube in this block?

S11: This is a face for example (*the student pointed at the correct area to show a face*).

R: OK. How many cubes in the block have 4 visible faces?

S11: None.

R: Let's look at the block a lit bit more closely. You were supposed to imagine that you were holding the block in your hands and you could rotate it as you wish.

S11: I still think none of them have 4 visible faces.

R: OK. Let's continue then. How many cubes in the block have 2 visible faces?

S11: 1,2... and 2 more in the other side; 2 plus 2 make 4 (*the student marked Cubes I and K*).

R: How many cubes in the block have no visible faces?

S11: 1... No not this one because it has a visible face. If we rotate the block (*the student pointed at the cubes in the central part*), we can see the lower part. So, the answer should be none.

R: Why?

S11: Because if there were 3 rows (*the student pointed at the cubes in the central part of the block*) we couldn't see those cubes in the center. But there are 2 rows here and that's why we see at least 1 face.

R: Can you draw the top view image of the block again?

S11: There are 4 here and 4 more here (*the student pointed at the 4 cubes in the front and 4 in the rear part*). There should be 2... No, I can't do it. I can't draw it exactly (*the student drew the block incorrectly; see Figure 19*).

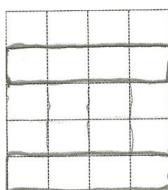


Figure 19. The top view image of block 4 drawn by S11.

S11 was able to successfully able to recognize cubes, faces, and edges. The student was able to count the number of the unit cubes systematically but gave incorrect answers to all of the other questions during the interview. We think this was probably because the student could not mentally visualize the block properly and detect the visible faces accurately. Also, when asked to draw the top view image of the block, the student's lines were very clear and strong for the cubes in the front and rear rows but unclear and weak for the cubes in the central part. In addition, the student came up with incorrect drawings for the different view images of the block. Evidently, the student couldn't draw pictorial representations properly.

The dialogue between the researcher and S20 (underachiever) regarding the questions about the Block of

Cubes Test block 6 (see Figure 16) and the researcher's notes are given below.

R: How many cubes in the block 6 have 4 faces? Can you show the face of a cube?

S20: This part is a face for example (*the student correctly showed a face*).

R: Can you find the cubes with 4 visible faces?

S20: I guess so. 1,2,3 and we could see the bottom face if we rotated the block (*the student marked Cube A*). There is this (*the student marked Cube I*). 1,2 and 3... 3 faces are visible but not the bottom, so this is not one of them. But that one is (*the student marked Cube B*). And maybe this one... it has 1,2,3 and 4 visible faces (*the student marked the edges of Cube J*).

R: Are those the faces?

S20: No. They are the edges. Right... this is not one of them.

R: Now, let's see the next question. How many cubes in the block have 3 faces?

S20: We have this... 1 and then 2,3,4,5,6,7 and 8. We could rotate the block... 9 and 10 and those with visible faces in the back... 11,12,13 and 14 in total.

S20 understood cubes but was unable to distinguish faces from edges. Similarly, the student made mistakes about the different numbers of the visible faces due to misconceptions about cubes and the student's inability to mentally visualize the block properly. The student initially counted the edges of some cubes instead of the faces but eventually realized the mistake. On the other hand, the student also made errors about the numbers of the visible faces of the cubes in the rear part.

Students using strategies like symmetry

In this subsection, the students who used strategies such as the use of symmetry to answer questions are discussed. The dialogue between the researcher and S9 (achiever) regarding the questions about the Block of Cubes Test block 3 (see Figure 20) and the researcher's notes are given below.

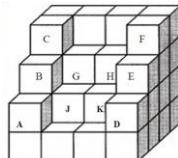


Figure 20. The Block of Cubes Test block 3.

R: So, how many cubes are in this block? How did you answer this question?

S9: 1,2,3 and 4 (*the student counted the four cubes in bottom row in the right end*). 1,2,3 and 4 (*the student counted the four cubes in bottom row in the front part*). 4 times 4 make 16, so there are 16 cubes in the bottom row. We've got 4 here (*the student counted the 4 cubes in the right end of the second row*). There are 4 here (*the student counted those in the left end of the second row*). It's 24 now. And 3 more here (*the student counted those in the front part of the second row*). There are 3 here (*the student counted those in the left end of the third row*). It's 30 now. Um, 2 more in the left end 2 more in the right end (*the student counted those in the left and right ends of the third row*). Now, we've got 34 cubes. There are 4 more in the rear part and 2 more here, so that makes 40 now (*the student counted the cubes in the middle part of the second row*). We have 4 more here and the number is 44 now (*the student counted the cubes in the middle part of the third row*). Um, when we add these two in the upper row, it now makes 46 in total.

R: Now, can you draw the rear view image of the block?

S9: There are 4 cubes here... that's how I see it. I should partly see some this side as well (*the student drew the block correctly; see Figure 21*).

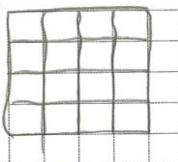


Figure 21. The rear view image of Block 3 drawn by S9.

S9 answered most of the questions in the Block of Cubes Test correctly, which shows that the student could visualize cubes and followed an appropriate and systematic way of thinking while answering the questions. The student made use of symmetry to find the number of the unit cubes in the block which means that the student recognized opposite sides of the block are the same. Similarly, the student came up with a correct rear view image drawing of the block. The answers given by the student about the other blocks indicate that the student gave correct answers using symmetry about the numbers of the unit cubes.

Discussions and Conclusion

The Block of Cubes Test results indicate that the sixth-grade students in this study had a low level achievement with regard to (a) finding the total numbers of unit cubes in the three-dimensional structures that are made from unit cubes, (b) finding the different numbers of the visible faces in these structures and (c) drawing the structure from a different viewpoint. The students in this study were introduced unit cubes in previous grades but they did not work with three-dimensional concrete models made from unit cubes. However, according to the primary education sixth grade mathematics curriculum, the students were supposed to be able to mentally visualize different-perspective views of three-dimensional structures made from unit cubes, to interpret two-dimensional pictorial representations of three-dimensional structures, and to find the visible faces of the cube.

Findings from the interviews with the students revealed that the average-achievers and underachievers couldn't distinguish between concepts like the face, the edge and the corner. We observed that some of the students counted the faces of the unit cubes as cubes and labeled the edges of the unit faces as faces. When asked the total number of unit cubes in the three-dimensional structures, the majority of them considered the block in two-dimension and only counted the cubes that were visible on paper. This shows that they had problems in mentally visualizing the pictorial representations of three-dimensional structures and answering the relevant

questions. Also, some of the students failed to follow a systematic way of counting when trying to find the numbers of unit cubes in the block. They either counted a cube more than once or forgot to count it at all, especially the hidden ones. Similarly, they performed incomplete counting about the questions asking the number of cubes with 1, 2 and 3 visible faces because they couldn't mentally rotate and visualize the blocks properly. With regard to the last question about the blocks, which asked the students to draw different view images of the blocks, the majority of the students came up with incorrect pictorial representations. In parallel with the findings of this study, Battista and Clements (1998), and Geddes and Fortunato (1993) stated that reasoning employed in finding the number of cubes in cuboids provides a cognitive framework for understanding the measurement of volume and the formula for determining the volume. However, the findings from relevant studies (Clements & Battista, 1992; Ben-Chaim et al. 1985) point out that children at primary education level have difficulty finding the number of unit cubes in cuboids. Hirstein (1981) claims that even the students in high schools have the same difficulty.

On the other hand, the students gave more correct answers to the questions about the blocks containing fewer cubes. Some of the students gave correct answers in the interview to the questions which they answered incorrectly in the Block of Cubes Test probably because they could think more carefully in the interview. Moreover, it was observed that some of the students who had higher number of correct answers got the total number of unit cubes in structures with symmetrical features by making use of symmetry. Similarly, Ben-Chaim, Lapan and Houang (1985) reported that students follow different methods to find the number of cubes in cuboids. According to their findings, the students in their study tried to find the number of cubes by counting the visible faces and then multiplying this number with two or counting twice and by counting visible cubes and then multiplying the number with two or counting twice. The students who missed the details about the block employed incorrect counting strategies. Also, some of the students considered the blocks as if they had been two-dimensional rather than three-dimensional and skipped the

hidden parts of the blocks. Finally, we concluded that the participants could not properly comprehend the two-dimensional representations of three-dimensional structures and mentally visualize the blocks appropriately.

With regard to the question which asked the students to draw different view images of the blocks, the majority of the students came up with incorrect pictorial representations. On the other hand, students with abilities to draw different-perspective views of blocks can easily recognize the faces of three-dimensional structures. These abilities make it easier for them to acquire the knowledge and skills to identify different-perspective views of prisms and calculate the volumes of prisms. Olkun (2003b) showed that primary school students' spatial abilities can be improved by means of activity samples designed to improve their spatial abilities with engineering drawing approaches. Children can better understand three-dimensional structures on paper when they work with concrete objects. Olkun and Sinoplu (2008) found that the toys made by fourth- and fifth-grade students by using unit cubes and triangular prisms improved their understanding of solid objects made from unit cubes. Their study showed that these toys, which they made from unit cubes by looking at the structures whose pictures were given, assisted students in discovering the spatial relationship among the elements of a structure as well as the relationships between concrete structures and drawings. Therefore, students should be given chances to work with concrete models initially so that their abilities to draw different-perspective views of objects can be improved. For students who have difficulty visualizing the blocks from the top, bottom, sides, front, and back, they can be asked to construct the blocks using same-size cubes. Students can then locate the cubes with the specified number of faces showing and students should be encouraged to consider one layer at a time.

We also found in this study that misconceptions were one of the primary causes of the students' low level of achievement in spatial visualization. Geometry courses should put more emphasis on geometric concepts. Chi (1992) attributes misconceptions mainly to (a) misleading or deficient understandings of the previously acquired concepts, (b)

difference between the functions of concepts in daily language and scientific language, (c) failure to create appropriate educational environments to teach subjects and concepts, and (d) failure to associate concepts with each other and daily life. If concepts such as face, edge and corner, which are significant for teaching three-dimensional geometry, are taught by giving detailed explanations and concrete examples in geometry lessons, they can be acquired by learners at different levels more efficiently. It may be a good idea to determine students' readiness level and eliminate deficiencies before teaching three-dimensional geometry so that a more efficient method of teaching geometry can be achieved.

Students' spatial visualization achievement level can be improved in light of the achievements covered by the primary education mathematics curriculum once they start to understand three-dimensional geometry. Also, linking new information to students' prior knowledge can bring about permanent learning. In this regard, teaching three-dimensional geometry can be built upon two-dimensional geometry. This can be useful in terms of both eliminating deficiencies in two-dimensional geometry and creating an opportunity for students to explore the differences between two-dimensional and three-dimensional geometry. It is vital that students realize the importance of the difference between three-dimensional and two-dimensional geometric blocks so that spatial visualization achievement can be improved.

Several researchers emphasize the importance of involving students in concrete spatial activities before forming and using their spatial imagination skillfully (Olkun & Sinoplu, 2008; Keller, Wasburn-Moses and Hart, 2002). Some spatial activities can be designed to improve students' spatial ability and in these activities, students may be asked to create concrete models with unit cubes. Then, they may be asked to find the total number of cubes or find the number of cubes with no visible faces in this model and explore the different view images of the block in order to correct any possible mistakes by rearranging or rotating these models. Therefore, the process of improving students' spatial visualization ability could be based on concrete models first. Also, Gutierrez (1992) stated that students should deal with three contexts while studying three-

dimensional geometry or objects: using concrete models that should be studied in a sequential order, using three-dimensional objects given in computer screen and drawing and reading pictorial representations. In addition to these activities, pictorial representation of a model made from unit cubes can be given to students on computer screen and students can first be asked to work with fixed imaged on screen. Then students may be guided through realizing their mistakes by means of the different view images of that model in a dynamic program. After that, students may be asked to try to think three-dimensionally by using the pictorial representations of the three-dimensional structure. After concrete model activity and activities performed with a dynamic computer application, students can more easily understand the pictorial representations of three-dimensional structures and therefore their spatial thinking ability can be improved.

The achievements covered by the primary education mathematics curriculum in Turkey highlight the importance of improving spatial ability. Also, it is expected that drawing different view images of structures that are made from unit cubes will help students learn to see things from different perspectives. According to data in the guide books published by the Turkish Ministry of National Education, very little time is allocated for having students acquire this ability. The subjects concerning the improvement of this ability should be spread across the curriculum and continuity of learning can be maintained. In addition, it is essential that students should realize the importance of attention and concentration for answering the questions requiring spatial ability. The spatial ability behaviors in this study were limited to the restructured primary education sixth-eighth grade mathematics curriculum in Turkey. In order to improve primary school students' spatial ability, some experimental studies could be conducted into the content and efficiency of activities and teaching methods that are suitable for those students with different learning paces.

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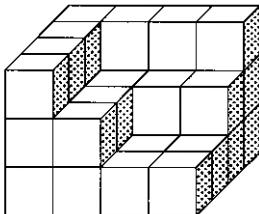
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Appendix

The Block of Cubes Test

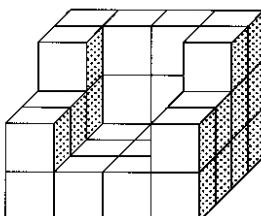
Cubes in the middle and top layers have cubes underneath them.
Imagine picking up the block and looking at the left face, bottom, and back.

Block 1



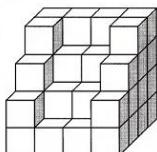
1. How many cubes are in the block 1?
2. How many cubes have 4 faces showing?
3. How many cubes have 3 faces showing?
4. How many cubes have 2 faces showing?
5. How many cubes have 1 face showing?
6. How many cubes have 0 faces showing?
7. Draw the right view images of the block 1.

Block 2



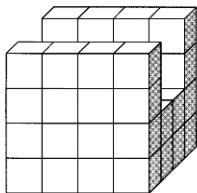
1. How many cubes are in the block 2?
2. How many cubes have 4 faces showing?
3. How many cubes have 3 faces showing?
4. How many cubes have 2 faces showing?
5. How many cubes have 1 face showing?
6. How many cubes have 0 faces showing?
7. Draw the left view images of the block 2.

Block 3



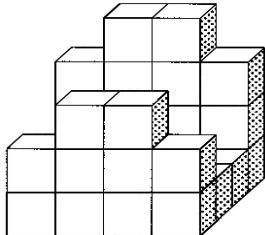
1. How many cubes are in the block 3?
2. How many cubes have 4 faces showing?
3. How many cubes have 3 faces showing?
4. How many cubes have 2 faces showing?
5. How many cubes have 1 face showing?
6. How many cubes have 0 faces showing?
7. Draw the rear view images of the block 3.

Block 4



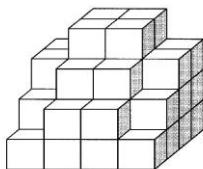
1. How many cubes are in the block 4?
2. How many cubes have 4 faces showing?
3. How many cubes have 3 faces showing?
4. How many cubes have 2 faces showing?
5. How many cubes have 1 face showing?
6. How many cubes have 0 faces showing?
7. Draw the top view images of the block 4

Block 5



1. How many cubes are in the block 5?
2. How many cubes have 4 faces showing?
3. How many cubes have 3 faces showing?
4. How many cubes have 2 faces showing?
5. How many cubes have 1 face showing?
6. How many cubes have 0 faces showing?
7. Draw the left view images of the block 5.

Block 6



1. How many cubes are in the block 6?
2. How many cubes have 4 faces showing?
3. How many cubes have 3 faces showing?
4. How many cubes have 2 faces showing?
5. How many cubes have 1 face showing?
6. How many cubes have 0 faces showing?
7. Draw the top view images of the block 6